Weakening cartography: On the formal foundation of functional hierarchies

Chenchen Song
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1 Introduction

Syntactic cartography (henceforth cartography) is a branch of generative syntax about the fine-grained hierarchical organization of functional categories. According to Shlonsky & Bocci (2019), its aim "is to draw maps of the structures of syntactic constituents, characterize their functional structure, and study the array and hierarchy of syntactically relevant features." The cartographic approach to natural language syntax grew out of generativists' interest in the 1990s in X'-style functional projections and their "splitting," such as Pollock's (1989) split-IP and Rizzi's (1997) split-CP. For recent overviews, see the above-mentioned Shlonsky & Bocci (2019) as well as Rizzi & Cinque (2016).

My goal in this short paper is to revisit the formal foundation of cartography from a mathematical order-theoretic perspective. My discussion is mainly conceptual, but I hope the results here can help prepare the ground for more empirical inquiries in future research. Following this introduction, I will first examine the assumptions of classical cartography in a formally explicit way (§2) and then discuss two of its design problems (§3). After that, I will review two existing studies attempting to "save" cartography by weakening its axioms (§4) and present a new proposal combining their main ideas (§5). Finally, I will briefly discuss the bigger picture of my proposal (§6).

2 Classical cartography, formally

By "classical cartography," I mean the framework established in the seminal works mentioned above, mainly those of Rizzi and Cinque. On the classical view, functional hierarchies are categorial sequences, as in (1).

(1) a. [Force [Top* [Int [Top* [Foc [... [Mood$_{\text{eval}}$ [Mood$_{\text{evid}}$ [Mod$_{\text{epis}}$ [Tense$_{\text{pst/fut}}$ [Mod$_{\text{nec}}$ [Aspect$_{\text{hab}}$ [... [Voice$_{\text{pass}}$ [**Verb**]]]]]]]]]]]]]]]

b. [(Integrated) nonrestrictive relative clauses [Universal quantifiers [Demonstratives [... [Numeral classifiers [... [Material AP [Classificatory APs [Proper NP

*The officially published version of the paper is almost identical to this draft, except for occasional rewording and omission in order to meet the page limit requirement.

[**Common NP**]]]]]]]]] (adapted from Rizzi & Cinque 2016)

Each categorial sequence of this sort is the extended projection of a lexical category. Thus, it is usually assumed that there is a cartographic hierarchy for each of the four major parts of speech: V, N, A, and P. Based on this assumption, we can work with the following formal definition:

Definition 1. Each functional hierarchy $\mathbf{FH}_{\mathcal{A}}$ is a sequence given rise to by a binary relation $\mathbf{R}_{\mathcal{A}}$ on the categories of a major part of speech $\mathcal{A}$.

The binary relation in question is usually taken to be functional selection. Thus, for two categories X and Y of the major part of speech $\mathcal{A}$, $\mathbf{R}_{\mathcal{A}}(X,Y)$ holds if and only if X functionally selects Y in syntactic derivation. This selection-based binary relation is not free but must obey the axioms below based on the assumptions of classical cartography:

(2) a. Irreflexivity: $\forall X \in \mathcal{A}$, $\neg\mathbf{R}_{\mathcal{A}}(X,X)$
(No category can select itself.)

b. Asymmetry: $\forall X,Y \in \mathcal{A}$, $\mathbf{R}_{\mathcal{A}}(X,Y) \Rightarrow \neg\mathbf{R}_{\mathcal{A}}(Y,X)$
(The selection between any two categories is fixed in direction.)

c. Transitivity: $\forall X,Y,Z \in \mathcal{A}$, $\mathbf{R}_{\mathcal{A}}(X,Y) \wedge \mathbf{R}_{\mathcal{A}}(Y,Z) \Rightarrow \mathbf{R}_{\mathcal{A}}(X,Z)$
(The binary relation is transitive.)

d. Totality: $\forall X,Y \in \mathcal{A}$, $\mathbf{R}_{\mathcal{A}}(X,Y) \vee \mathbf{R}_{\mathcal{A}}(Y,X)$
(No category is excluded from the hierarchy of its major part of speech.)

These axioms together make a cartographic hierarchy into a *strict total order*. In particular, transitivity has been heavily relied on in the development of classical cartography, irreflexivity is self-evident, and totality has always been taken for granted. Asymmetry requires a bit more clarification, since flexibly positioned categories have been observed since the early days of cartography, such as the iterating Top* in (1a). However, the asymmetry axiom can be maintained to the extent that closer examination can reveal subtle syntacticosemantic distinctions between iterating categories, in the same way as the multiple Split-IP categories in (1a) are assigned distinctive subscripts. For instance, Benincà & Poletto (2004) argue that the multiple Top*s above are in fact nonidentical.

3 Design problems of classical cartography

Classical cartography is problematic in design in multiple aspects. In this section, I focus on two most serious problems: transitivity failure (§3.1) and totality failure (§3.2).

3.1 Transitivity failure

Transitivity failure is a problem of classical cartography that has been repeatedly brought up in the literature. This failure occurs when given categories X,Y,Z of a major part of speech $\mathcal{A}$, $\mathbf{R}_{\mathcal{A}}(X,Y)$ and $\mathbf{R}_{\mathcal{A}}(Y,Z)$ do not necessarily lead to $\mathbf{R}_{\mathcal{A}}(X,Z)$. For example, Nilsen (2003) observes that in Norwegian, while the adverbs *muligens* 'possibly' and *alltid* 'always' respectively precede and follow the negation adverb *ikke* 'not', they can appear in the reversed order between themselves, as in (3a–c). This situation is formally represented in (3d).

(3) a. *Ståle har* ***muligens ikke*** / ***ikke muligens** *spist* [Norwegian]
S has possibly not eaten

hvetekakene sine.
the-wheaties his
'Stanley possibly hasn't eaten his wheaties.'

b. *Ståle hadde* ***alltid ikke** / ***ikke alltid*** *spist hvetekakene sine.*
S had not always eaten the-wheaties his
'Stanley hadn't always eaten his wheaties.'

c. *Dette er et morsomt, gratis spill hvor spillerne* ***alltid muligens*** *er*
this is a fun free game where the-players always possibly are

et klikk fra å vinne $1000!
one click from to win $1000
'This is a fun, free game where you're always possibly a click away from winning $1000!' (Nilsen 2003: 10–11)

d. $\mathbf{R}_{\mathcal{V}}(\mathbf{H}(\text{possibly}), \text{Neg}) \wedge \mathbf{R}_{\mathcal{V}}(\text{Neg}, \mathbf{H}(\text{always})) \wedge \mathbf{R}_{\mathcal{V}}(\mathbf{H}(\text{always}), \mathbf{H}(\text{possibly}))$
($\mathbf{H}(e)$ is the head of the projection hosting the expression e, say, as its Spec)

Similarly, van Craenenbroeck (2006) observes that in Venetian, while embedded *wh*-phrases and phrases that have gone through clitic left dislocation (CLLD) respectively precede and follow the complementizer *che* 'that', they can only appear in the reversed order between themselves regardless of the position of the complementizer, as in (4a–c). Assuming that *wh*-phrases, *che*, and CLLD-ed phrases are respectively hosted by some Focus, C, and Topic projections, we can formally represent this situation by the statement in (4d).

(4) a. *Me domando* ***chi che*** / ***che chi** *Nane ga visto al marcà.* [Venetian]
me I.ask who that Nane has seen at.the market
'I wonder who Nane saw at the market.'

b. *Me dispiase* ***che a Marco*** / ***a Marco che** *i ghe gabia ditto*
me is.sorry that to Marco they to.him have.SUBJ told

cussi.
so
'I am sorry that they said so to Marco.'

c. **Me domando* ***a chi*** *(che)* ***el premio Nobel*** *(che) i ghe lo podarà*
me I.ask to who that the prize Nobel that they to.him it could

dar.
give
Intended: 'I wonder to whom they could give the Nobel Prize.'
(van Craenenbroeck 2006: 53–54)

d. $\mathbf{R}_{\mathcal{V}}(\text{Focus}, \text{C}) \wedge \mathbf{R}_{\mathcal{V}}(\text{C}, \text{Topic}) \wedge \mathbf{R}_{\mathcal{V}}(\text{Topic}, \text{Focus})$

An additional case of transitivity failure is that in the split-IP domain of Imbabura Quechua, which is reported in Bruening (2019). In this head-final language, while the desiderative suffix *-naya-* and the progressive suffix *-ju-* respectively precede and follow the first-person suffix *-wa-*, they can appear in two different orders themselves, as in (5a–b). Assuming that the three morphemes respectively head three projections DesP, Agr_1P, and ProgP, we can formally represent this situation by the statement in (5c).

(5) a. *miku-**naya-wa-ju**-n* [Imbabura Quechua]
eat-DES-1-PROG-3
'I was wanting to eat.'

b. *miku-**ju-naya-wa**-n*
eat-PROG-DES-1-3
'I wanted to be eating.' (adapted from Bruening 2019: 4)

c. $\mathbf{R}_{\mathcal{V}}(\text{Prog}, \text{Agr}_1) \wedge \mathbf{R}_{\mathcal{V}}(\text{Agr}_1, \text{Des}) \wedge \mathbf{R}_{\mathcal{V}}(\text{Prog}, \text{Des}) \wedge \mathbf{R}_{\mathcal{V}}(\text{Des}, \text{Prog})$

Note that due to the head-finality of Imbabura Quechua, the linear affixal orders in (5a–b) are the mirror image of the selection-based binary relation instances in (5c).

One could potentially argue away some or even all of the documented cases of transitivity failure by resorting to additional derivational means (e.g., van Craenenbroeck 2006) or a more dynamic view of syntactic derivation (e.g., Zwart 2009). But the problem of the transitivity axiom is arguably more than just counterexamples. Its deeper trouble, which cannot be argued away, is that selection itself is not a transitive relation. This is clearly reflected in the Imbabura Quechua case above, where *-ju-naya-*, *-naya-wa-*, and *-ju-naya-wa-* are all allowed, but not **-ju-wa-*. This means that while **H**(*-wa-*) selects **H**(*-naya-*) and **H**(*-naya-*) selects **H**(*-ju-*), **H**(*-wa-*) does not select **H**(*-ju-*). If selection itself is nontransitive, the binary relation defined by it cannot be transitive either.

Related to the above is the "problem of plenitude," as Larson (2021) puts it. Due to the inherent nontransitivity of functional selection, cartographic hierarchies can only exist in their full forms, with no omissible or skippable categories. But this gives rise to a plenitude of empty, uninterpreted categories in most concrete derivations. Larson illustrates this with the phrase *large wide board*, which must have the verbose structure in (6a) rather than the truncated structure in (6b).

(6) a. $[_{\text{sizeP}}$ large $[_{\text{lengthP}}$ $[_{\text{heightP}}$ $[_{\text{speedP}}$ $[_{\text{depthP}}$ $[_{\text{widthP}}$ wide $[_{\text{NP}}$ board]]]]]]]

b. *$[_{\text{sizeP}}$ large $[_{\text{widthP}}$ wide $[_{\text{NP}}$ board]]] (adapted from Larson 2021: 249)

Given the empirical commonality of transitivity failure and the counterminimalist nature of the problem of plenitude, the most natural conclusion to draw here is that either the transitivity axiom is wrong, or the selection-based definition of the binary relation **R** is.

3.2 Totality failure

While previous concerns about the formal foundation of cartography mostly target the transitivity axiom, Song (2019: Chapter 5) further notices that the totality axiom in classical cartography is also problematic, based on the observation that some categories belong to the same functional hierarchy but never co-occur by design and hence cannot be part of the binary relation defining their ambient hierarchy.

A familiar scenario of this sort is the alternation between ϕ-complete and defective categories in Chomsky (2001), such as T_{comp} vs. T_{def} and v_{comp} (=v^*) vs. v_{def} (=v). A ϕ-complete category and its defective counterpart cannot co-occur in the same projection line—that is, without functional hierarchy–restarting strategies like subordination. See (7) for an illustration (for expository convenience I omit the subscript "comp" for ϕ-complete categories).

(7) a. [$_{\text{TP}}$ the committee T [$_{v^*\text{P}}$ v^* awarded several prizes]]
b. [$_{\text{TP}}$ several prizes$_i$ T [$_{v\text{P}}$ are awarded t$_i$]]
c. [$_{\text{TP}}$ several prizes$_i$ T [$_{v\text{P}}$ are likely [$_{\text{T}_{\text{def}}\text{P}}$ to [$_{v\text{P}}$ be awarded t$_i$]]]]
(based on Chomsky 2001: 7)

As we can see, only one of v^* (ϕ-complete) and v (defective) can appear in a simple monoclausal structure like that in (7a) or (7b). In the biclausal structure in (7c), there are both T and T_{def}, but these are in two separate projection lines, one in the matrix clause and the other in the infinitival clause. Thus, for any category, only one of its ϕ-complete and defective versions can be fit into a classical cartographic hierarchy.

Another counter-totality scenario in minimalist syntax involves "flavored" categorizers, in the sense of Distributed Morphology (Halle & Marantz 1993 et seq.). See (8) for some examples.

(8) a. Folli & Harley (2005): v_{DO}, v_{CAUSE}, v_{BECOME}
b. Lowenstamm (2008): n_{I} (MASC), n_{II} (FEM), n_{III} (NEU), n_{IV} (other)

To the extent that these are bona fide categorizers—namely, functional categories that merge with and categorize roots—they cannot co-occur in the same projection line, since each root can only be categorized once in the same categorization cycle or workspace.[1] This situation is clearer in (8b), for a noun can only be of a single gender in any specific derivation. Take German for example.

(9) [$_{\text{N}_{\text{MASC}}}$ n_{I} √ZUG] 'train', [$_{\text{N}_{\text{FEM}}}$ n_{II} √WAND] 'wall', [$_{\text{N}_{\text{NEU}}}$ n_{III} √BUCH] 'book'

Some German nouns have more than one gender, with different senses, but even those nouns can only have a single gender/sense in a specific use. For instance, it is impossible to use *See* simultaneously as masculine (meaning 'lake') and feminine (meaning 'sea'). Thus, the four flavors of n in (8b) are in strictly complementary distribution and cannot co-exist in the same classical cartographic hierarchy.

Things are less clear in (8a), since the various little vs are often not used as true categorizers (in that they do not categorize roots) in the literature but merely employed to introduce eventuality layers (see, e.g., Cuervo 2003). Song (2019: 164) calls this the "dummy verbalizer pitfall." Such eventuality-introducing categories *can* be fit into the same functional hierarchy, but then "categorizer" becomes a misnomer, and an alternative model like that in Ramchand (2008) is methodologically preferable.[2]

In sum, however the binary relation **R** for a cartographic functional hierarchy is defined, it should have room for alternating categories like the above. Formally speaking, such categories are *incomparable elements* in a binary relation:

(10) Nontotality: $\exists X, Y \in \mathcal{A}, \neg\mathbf{R}_{\mathcal{A}}(X, Y) \wedge \neg\mathbf{R}_{\mathcal{A}}(Y, X)$
(X and Y are incomparable)

[1] On this view, recategorization scenarios like *category*$_{\text{N}}$-*ize*$_{\text{V}}$-*er*$_{\text{N}}$ necessarily involve multiple cycles.
[2] Ramchand simply calls the eventuality-introducing layers Init, Proc, and Res, without using the term "categorizer" at all.

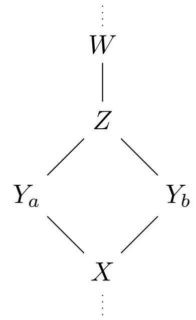

Figure 1: A functional hierarchy with flavored categories (Song 2019: 39)

4 Saving by weakening

Since both design problems mentioned above are about the nature of the binary relation underlying functional hierarchies, to find solutions we can revisit the binary relation itself. And given the shared bane of the two failures—namely, some axiom is too restrictive—the revisiting in question should be some sort of weakening. Two attempts have been made in the literature to "save" cartography in this way. I briefly review them in this section.

4.1 Song (2019): partial order

Song (2019) weakens the binary relation from a strict total order to a *partial order*.

Definition 2. A partial order $\leq$ on a set P is a binary relation contained in $P \times P$, such that

- $\forall p \in P$, $p \leq p$ (reflexivity),
- $\forall p, q, r \in P$, if $p \leq q$ and $q \leq r$, then $p \leq q$ (transitivity),
- $\forall p, q \in P$, if $p \leq q$ and $q \leq p$, then $p = q$ (antisymmetry).

Comparing these axioms with those in (2), we can see that Song (2019) has removed totality, toggled irreflexivity, and changed asymmetry to antisymmetry. Apart from the third move, which is not triggered by the problems in §3 but is a concomitant of the partial order view itself (and in effect bans ordering cycles from functional hierarchies), both the first and the second move directly address the problems in §3.

The removal of totality is meant to allow cartographic hierarchies to accommodate incomparable categories, as illustrated in Figure 1, where X, Y, Z, and W are categories, and the subscripts a and b mark two complementary flavors of Y. As we can see, both Y_a and Y_b are normally ordered with respect to other categories in the hierarchy, yet they are unordered with respect to each other. Importantly, this scope-based hierarchy should be understood as a structure in the ontology of syntactic categories rather than a syntactic object assembled in concrete derivations. This shift of perspective is key to Song's (2019) model.

The toggling of irreflexivity also follows from said perspective shift, which is more exactly a change in the defining criterion for the binary relation underlying cartographic functional hierarchies—from a selection-based perspective to a scope-based one.

Definition 3. (based on Song 2019: 146) For any categories X, Y of a major part of speech $\mathcal{A}$, if Y *functionally selects* X in syntactic derivation, then X *can fall in the functional selectional scope of* Y in the background ontology of syntactic categories, written $X \sqsubseteq Y$. The latter criterion defines functional hierarchies.

The notation $\sqsubseteq$ can be read "has a scope smaller than or equal to." The change of perspective may sound like a mere rewording, but it frees us from the shackles of selection. First, since any category has a scope (smaller than or) equal to itself, $\sqsubseteq$ is naturally reflexive. Second, since scoping is just an ontological concept but not a derivational operation (unlike selection), it is safely transitive and free from the problem of plenitude. Thus, the structure in (6b), repeated below as (11), is perfectly allowed in a scope-based version of cartography.

(11) $[_{\text{SIZEP}}$ large $[_{\text{WIDTHP}}$ wide $[_{\text{NP}}$ board $]]]$

However many categories there are between SIZE and WIDTH in the adjectival hierarchy, the statement WIDTH $\sqsubseteq$ SIZE (WIDTH has a scope smaller than or equal to SIZE) independently holds, without the mediation of those intervening categories.

As mentioned above, the key feature of Song's (2019) model is the explicit separation of derivational and ontological issues in syntactic theory. Another feature of this model is that it has a unified defining criterion ($\sqsubseteq$) for all **R**s, with the different cartographic hierarchies merely differing in the major part of speech they belong to. In addition, each **R** in this model is defined for an entire cartographic hierarchy.

4.2 Larson (2021): total preorder

While Song's (2019) model still largely keeps to the basic format of classical cartography, Larson's (2021) model deviates from that format to a much greater degree. Larson shifts the locus of the order relations underlying cartographic hierarchies from syntactic categories to features, which do not project their own heads but are collectively borne by a few pivotal heads (e.g., C, D). Each such collection of features is equipped with a *total preorder*, which is again weaker than the strict total order in classical cartography.

Definition 4. A total preorder $\leq$ on a set P is a binary relation contained in $P \times P$, such that

- $\forall p \in P$, $p \leq p$ (reflexivity),
- $\forall p, q, r \in P$, if $p \leq q$ and $q \leq r$, then $p \leq q$ (transitivity),
- $\forall p, q \in P$, $p \leq q$ or $q \leq p$ (totality).

Larson's toggling of the irreflexivity axiom in classical cartography also follows from a change in the defining criterion for the order relation. Specifically, he also abandons the selection-based view in favor of a safely transitive criterion (such that no problem of plenitude arises). But unlike Song, who merely redefines selection as selectional scope comparison, Larson leaves the ordering criterion open to variation and relativizes it to each cartographic zone (e.g., CP, IP). For instance, the ordering criterion for the adjectival zone is cognitive subjectivity (à la Scontras, Degen & Goodman 2017): the less subjective an adjective is, the closer it is to the head noun, and so the lower it is in its ambient cartographic hierarchy. See (12) for an illustration.

(12) $\text{D}_{\{\ldots([\textsc{color}]/[\textsc{material}],\ [\textsc{size}])\ldots\}}$ (adapted from Larson 2021: 257/262)

Larson (2021) uses the parenthesis notation (a, b, c) for the preorder $a \leq b \leq c$ and uses the slash notation a/b for the bidirectional ordering $a \leq b \wedge b \leq a$, which is made possible by the absence of asymmetry/antisymmetry in this model. The ordered feature set in (12), which Larson calls a "proset," gives rise to an actual adjectival sequence (e.g., *a small furry gray mouse*) by a series of derivational steps involving D and its light counterpart d, which are procedurally ordered by the proset. I abstract away from the technical details due to space limitations. See (13) for another example.

(13) $\text{E}_{\{\ldots([\textsc{fin}],\ [\textsc{top}]/[\textsc{foc}],\ [\textsc{force}])\ldots\}}$ (adapted from Larson 2021: 264)

These are the split-CP categories from Rizzi (1997), recast in Larson (2021) as features in a proset borne by the pivotal category E (for "expression"), which Larson uses (following Banfield 1973) instead of the conventional label C. This proset-bearing E, together with its light counterpart e, gives rise to the cartographic sequence of left-periphery elements. Note that while the cartographic features themselves live in some fixed-length orders in the background ontology, the actual prosets occurring in concrete derivations are not invariant. Although Larson does not make this fully clear, what feature is included and what is not is presumably a matter of lexical selection (at the lexical array–forming stage). What matters for the model is that whatever features selected into the prosets fall in their predetermined order in the ontology.

A major advantage of Larson's (2021) model, which distinguishes it from both classical cartography and Song's (2019) model, is that it has room for some bona fide cases of transitivity failure—that is, cases of flexible ordering that cannot be argued away by derivational means, such as the existence of both color≺material and material≺color in the adjectival zone (e.g., *a furry gray mouse* and *a gray furry mouse*).[3] As mentioned above, Larson's solution is to allow cycles in the order relation by removing the asymmetry axiom (and not introducing antisymmetry). However, like classical cartography, Larson's model has no room for incomparable categories, probably because those categories are not his empirical focus. And due to the lack of a unified ordering criterion, it might actually encounter difficulty in finding appropriate cognitive factors to define the miscellaneous feature prosets. For instance, Larson does not specify what the ordering criterion in (13) is but merely assumes its existence.

5 A middle-way proposal

The shared merit of Song's (2019) and Larson's (2021) weakening of classical cartography is that both models are freed from the "selection pitfall" described in §3. Two direct consequences of this merit are the transitivity and the reflexivity axiom. However, the two models also each have their disadvantages. Song's (2019) model has room for incomparable elements but not for truly flexibly ordered elements, and the opposite is true for Larson's (2021) model. If possible, we want to have the best of both worlds, and that is what I will propose below.

[3]Larson (2021) also treats the flexible ordering of Topic and Focus as a case of true flexible ordering, hence the slash notation in (13). However, as Larson points out in his footnote 17 (p.263), this is a debatable issue.

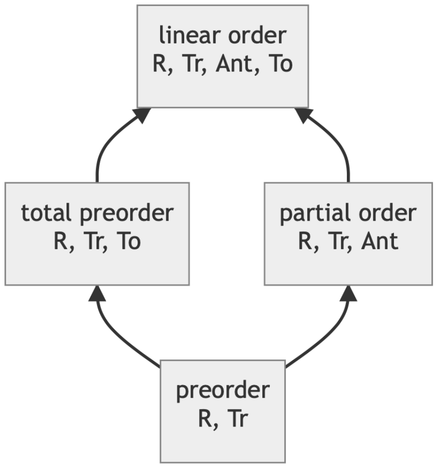

Figure 2: Four order relations ordered by their "strength"
(R = reflexive, Tr = transitive, To = total, Ant = antisymmetric)

Definition 5. Weak cartographic hypothesis (WCH) All functional hierarchies are *preorders*. Some of them are furthermore total preorders, partial orders, or linear orders.

The above definition utilizes the "strength" relation between various order relations, as in Figure 2. From the bottom up, the weakest kind of order relation is just a plain preorder (reflexive, transitive). There are two ways to strengthen a preorder, either by making it total or by making it partial (via banning cycles). Finally, we can make both order relations even stronger by combining their properties and getting a linear order (aka total order or chain). The formal definitions of these order relations can be found in any introduction to mathematical order theory (e.g., Schröder 2016).

For simplicity's sake, I follow classical cartography and Song (2019) and impose the order relations thus defined on categories, but a Larsonian, feature-based implementation is also imaginable. On the weakened definition of cartography, what distinguishes the category-based and the feature-based implementation is no longer their handling of the problems in §3—since both can handle them—but factors from other dimensions, such as economy.

On the WCH, functional hierarchies may take any of the four forms below. As usual, I use capital letters $X, Y, Z, \ldots$ to denote syntactic categories. And for expository convenience, I write $X \to Y$ for $X \sqsubseteq Y$ and use $\{X, Y\}$ to mean that X and Y are incomparable.

1. The chain (i.e., linear order):

$$\ldots X \to Y \to Z \to W \to V \ldots$$

2. The connected directed graph or digraph, with incomparable elements (i.e., preorder):

$$\ldots X \to Y \leftrightarrows Z \to \{W_1, W_2\} \to V \ldots$$

3. The connected digraph, without incomparable elements (i.e., total preorder):

$$\ldots X \to Y \leftrightarrows Z \to W \leftrightarrows V \ldots$$

4. The directed acyclic graph or DAG (i.e., partial order):

$$\ldots X \to \{Y_1, Y_2, Y_3\} \to Z \to \{W_1, W_2\} \to V \ldots$$

Functional (sub)hierarchies are typically chains, especially if we strive for a highly fine-grained level of description, with the subtle differences between alleged iterable categories being taken into account (as in Benincà & Poletto 2004). Hence, the classical view is fine in many or even most cases, and linguists whose immediate concerns are order-theoretically nonexceptional (i.e., with no incomparable categories or bona fide ordering cycles) may conveniently stick to classical cartography. It is only when the empirical domain at hand manifests exceptional ordering patterns that the WCH becomes truly useful.

6 The bigger picture

In this paper, I examined the formal foundation of cartography from an order-theoretic perspective. Cartographic functional hierarchies in their classical conception are strict total orders. But this classical view is flawed and suffers from multiple problems, such as transitivity failure and totality failure. Song (2019) and Larson (2021) have attempted to free cartography from these problems by weakening its underlying order relation, respectively to partial orders and total preorders. My proposal in this paper (i.e., the weak cartographic hypothesis) is an eclectic combination of these two ideas.

So far, we have been focused on individual functional hierarchies. But the WCH furthermore supports a big-picture organization of the entire categorial inventory. Consider the two hierarchies in (14), which are respectively defined by the order relations $\mathbf{R}_{\mathcal{A}}$ and $\mathbf{R}_{\mathcal{B}}$, with $\mathcal{A}$ and $\mathcal{B}$ being two major parts of speech.

(14) a. $\mathcal{A} : \ldots X \to \{Y_1, Y_2\} \to Z \to W \ldots$
b. $\mathcal{B} : \ldots X \leftrightarrows Y \to Z \to W \ldots$

Assuming the omitted parts of the two hierarchies also conform to the patterns displayed in (14), $\mathcal{A}$ and $\mathcal{B}$ are respectively a partial order and a total preorder. But since both types of order relations are just strengthened preorders (see Figure 2), $\mathcal{A}$ and $\mathcal{B}$ by definition are still preorders. The same is true for all four possible forms of functional hierarchies in §5. This state of affairs leads to a nice big-picture view of functional hierarchies:

Definition 6. The various functional hierarchies of a language can join into a single preorder, which may be called a "superhierarchy."

This big-picture unification only works if all functional hierarchies share a single ordering criterion. Thus, between Song's (2019) and Larson's (2021) model, it is only compatible with the former, where the uniform ordering criterion is functional selectional scope. With this superhierarchical view, we can continue to formalize cartography at higher orders. For instance, we can now study the order-theoretic connections (e.g., monotone functions) across functional hierarchies. Song (2019: Chapter 6) explores this direction with the aid of mathematical category theory (Eilenberg & Mac Lane 1945 et seq.).

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